# A Bayes Space Approach to Profile Monitoring of Probability Density Functions for Image Data

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**Abstract**

The availability of advanced monitoring technologies is leading to data-rich industrial environments, where image data play a key role for statistical quality control. In this framework, the quality of the monitored part is often related to the overall distribution of relevant statistical descriptors rather than to the measured value for one single feature (e.g., their mean or variance). The latter distribution can be represented in the form of a probability density function (PDF), accessed from image data whenever the spatial autocorrelation is negligible (e.g., in random porous structures), and then used as a quality signature to determine the stability of the process.
Although control charting schemes for unconstrained functional data have been discussed in the literature (Woodall *et al.*, 2004 and Noorossana *et al.*, 2012), their use on density data yields non-optimal results, as these methods cannot properly handle the data constraints. In this work we propose a profile monitoring approach suitable for density functions, that is based on the theory of Bayes Hilbert spaces (Egozcue et al., 2006, van den Boogaart et al, 2014). We ground our developments on the simplicial functional principal components analysis (SFPCA) of Hron et al (2016), that allows to effectively cope with the data dimensionality, by properly accounting for their relative nature. We show through simulations that a Bayes space approach allows enhancing the monitoring performances with respect to those functional approaches that neglect the inherent features of density functions. We illustrate the proposed methodology through a real case study dealing with the quality control of foamed materials production.

## References

van den Boogaart, K.G., Egozcue, J.J., Pawlowsky-Glahn, V. (2014). Bayes Hilbert spaces. *Australian & New Zealand Journal of Statistics 56* (2), pp. 171-194.

Egozcue, J.J., Dìaz-Barrero, J.L., Pawlowsky-Glahn, V. (2006). Hilbert space of probability density functions based on Aitchison geometry. *Acta Mathematica Sinica (English Series) 22* (4), pp. 1175-1182.

Hron, K., Menafoglio, A., Templ, M., Hruzova, K., Filzmoser, P. (2016). Simplicial principal component analysis for density functions in Bayes spaces. *Computational Statistics and Data Analysis 94*, pp. 330-350.

Noorossana R., Saghaei A., Amiri A. (2012). *Statistical Analysis of Profile Monitoring*, John Wiley & Sons

Woodall, W.H., Spitzner, D.J., Montgomery, D.C., Gupta, S. (2004). Using Control Charts to Monitor Process and Product Quality Profiles. *Journal of Quality Technology*, 36(3), 309 - 320