Subcompositional coherence and the compositional complex

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Aitchison (2001) defines a compositional problem as one in which only the relative sizes of the multivariate sample components are relevant. Although formulated in terms of examples, the principle of subcompositional coherence is fundamental to the analysis of relative abundance data. In this paper, the notion of a compositional complex is introduced to provide a geometric interpretation of the principle of subcompositional coherence.

The complex is a combinatorial reconstruction of the closed n-dimensional simplex from all its faces, Gelfand and Manin (2003). Essentially, it is a collection of faces of the closed simplex and maps between these faces, also called face maps. The criterion of compatibility with these face maps is then defined to be the property of subcompositional coherence for a family of distributions on these faces.

It is shown how to interpret the subcompositional coherence property for the family of logistic normal distributions using this complex structure. For a family of distributions that has moments, the geometric criterion is interpreted in terms of the characteristic functions for such a family. We use this criterion to demonstrate that any distribution induced by the family of elliptically symmetric distributions using the additive log ratio (alr) provides subcompositionally coherent models. Similarly, this criterion also allows us to infer the coherence property for distributions that transform to the skew normal family of distributions Azzalini and Capitanio (1999) under alr, thus substantially expanding the class of distributions available on the simplex for modeling relative abundance data.

References

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